Direction of Arrival Based Spatial Covariance Model For Blind Source Separation[1]

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EE627A: Project Presentation
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April 10, 2016

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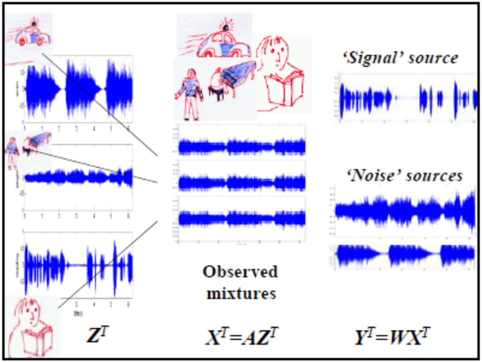
BSS(@Clifford)



ICA(Sawada et al.,2004)[5] and (Hyvrinen et al.,2001)[6]*[1]

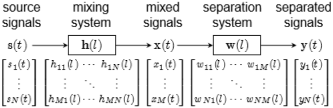


Fig. 1. BSS for convolutive mixtures

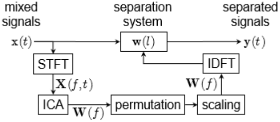


Fig. 2. Flow of frequency-domain BSS

- frequency-domain BSS involves a permutation problem:
  - the permutation ambiguity of ICA in each frequency bin should be aligned so that a separated signal in the time-domain contains frequency components of the same source signal.

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DOA(Ikram et al.)[7]and(Nesta et al.)[9]*[1]

- Source permutation is usually solved based on time difference of arrival (TDoA) interpretation of ICA mixing parameters.
- Phase differences become ambiguous when the frequency exceeds the spatial aliasing limit, which corresponds to a wavelength greater than half of the microphone spacing.
- As a result, the TDoAs cannot be directly utilized in solving the permutation problem for high frequencies.
- TDoA with the help of DUET [10] and binwise clustering [11].

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PCA and SVD (@Iyad Batal)

Objective: find the principal components P of a data matrix A(n,m).

1. First zero mean the columns of A (translate the origin to the center of gravity).
2. Apply PCA or SVD to find the principle components (P) of A.

PCA:

I. Calculate the covariance matrix C= $\frac{A\,A^T}{n}$

II. p = the eigenvectors of C.

III. The variances in each new dimension is given by the eigenvalues.

SVD:

I. Calculate the SVD of A.

II. P = V: the right singular vectors.

III. The variances are given by the squaring the singular values.

3. Project the data onto the feature space. F = P x A
4. Optional: Reconstruct A' from Y where A' is the compressed version of A.

What can matrix represent

- System of equations
- User rating matrix
- Image
- Matrix structure in graph theory
  - Adjacent matrix
  - Distance matrix

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- LU decomposition
  - Solving system of equations
- Singular Value Decomposition(SVD)
  - Low rank matrix approximation
  - Pseudo-inverse
- Probabilistic Matrix Factorization(PMF)
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- Non-negative Matrix Factorization(NMF)
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NMF (Paatero and Tapper, 1994)

Lee and Seung alternate updates to W and H^T using an ascent direction.

Observe: $\min \|A - WH^T\|_F$ is linear least squares for either W or H if the other is fixed.

Leading singular vectors of a nonnegative matrix are nonnegative. (Perron-Frobenius Theorem).

Immediately suggests a simple rank-1 NMF:

Compute $[U\ \Sigma\ V] = SVD(A)$

$$W = U_1,\ H = \Sigma_{11} V_1^T$$

NMF (@Author)*[1]

matrix $[u, \sigma, v] = \text{powermethod}(A)$:

$v = $ vector of ones

while not converged

$$u = \frac{Av}{\|Av\|}$$

$$v = \frac{A^T u}{\|A^T u\|}$$

$$\sigma = \|A^T u\|$$

end

[1] Credit Goes to Author

NMF (@Author)*[1]

$[W,H] = \text{nmf}(A)$:

1 for $i = 1:k$

2 $\quad [u,\sigma,v] = \text{powermethod}(A)$

3 $\quad W_i = u,\ H_i^T = \sigma v^T$

4 $\quad A = A - u\sigma v^T$

5 $\quad\quad$ for all $A_{i,j} < 0$ set $A_{i,j} = 0$

6 $\quad$ end for

- Without step 5, this will simply compute the SVD

[1] Credit Goes to Author

NMF (Paatero and Tapper, 1994)

Given $A \in {\mathbb{R}_+}^{m \times n}$ and a desired rank $k << min(m,n)$,
find $W \in {\mathbb{R}_+}^{m \times k}$ and $H \in {\mathbb{R}_+}^{k \times n}$ s.t. $A \approx WH$.

- $\min_{W \geq 0, H \geq 0} \|A - WH\|_F$
- NMF improves the approximation as k increases:
  If $rank_+(A) > k$,

  $$\min_{W_{k+1} \geq 0, H_{k+1} \geq 0} \|A - W_{k+1}H_{k+1}\|_F < \min_{W_k \geq 0, H_k \geq 0} \|A - W_k H_k\|_F,$$

  $W_i \in {\mathbb{R}_+}^{m \times i}$ and $H_i \in {\mathbb{R}_+}^{i \times n}$
- But SVD does better: if $A = U \Sigma V^T$, then
  $\|A - U_k \Sigma_k V_k^T\|_F \leq min\|A - WH\|_F$, $W \in {\mathbb{R}_+}^{m \times k}$ and $H \in {\mathbb{R}_+}^{k \times n}$
- So Why NMF? for Nonnegative Data
  NMF provides Better Interpretation of Lower Rank Approximation

NMF ([12,14,15,16,17,20])*[1]

- NMF have been proposed for separation of sound sources both with single and multichannel mixtures.
- In the NMF separation framework the spatial properties of the sources can be modeled using a spatial covariance matrix (SCM) for each source at each STFT frequency bin [18][22].
- Such extensions are hereafter referred to as complex-valued NMF (CNMF).
- SCM denotes the mixing of the sources by magnitude and phase differences between the recorded channels, and is not dependent on the absolute phase of the source signal.

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- Array capture consists of a mixture of sound sources convolved with their spatial responses.
- $$\tilde{x}_m(t) = \sum_{k=1}^{K} \sum_{\tau} h_{mk}(\tau)\, s_k(t-\tau)$$
- Which can be approximated as,

$$X_{il} \approx \sum_{k=1}^{K} h_{ik} s_{ik} = \sum_{k=1}^{K} y_{ilk}$$

where ,$\hat{x}_{il}$ of the capture $X_{il} = [X_{il1}, \ldots, X_{ilM}]$

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- SCM calculated at each time-frequency point.
  - Magntiude-square root version of array capture.

$$\hat{\mathbf{x}}_{il} = [|x_{il1}|^{1/2}\text{sign}(x_{il1}), \ldots, |x_{ilM}|^{1/2}\text{sign}(x_{ilM})]^T$$

- Abs phase of signal transformed to phase difference between microphone pair

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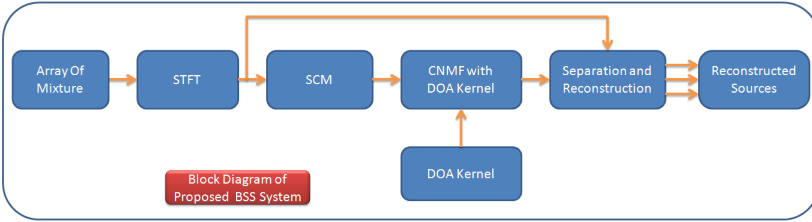
1. Mixing in Spatial Domain.
2. defined by, replacing each term by its covariance counter-part .

$$\mathbf{X}_{il} \approx \sum_{k=1}^{K} \mathbf{H}_{ik} \hat{s}_{ilk} = \sum_{k=1}^{K} \mathbf{S}_{ilk}, \qquad (5)$$

Basic Idea:

- BSS via estimation of source SCM of STFT of mixture signal.
- SCM model is combined with a parameter estimation is formulated in a complex-valued non-negative matrix factorization (CNMF) framework

Block diagram of Proposed Algorithm:



Time-difference of Arrival:

$$\tau_n(\mathbf{k}_o) = \frac{-\mathbf{k}_o^T(\mathbf{n}-\mathbf{p})}{v} = \frac{-\mathbf{k}_o^T\mathbf{n}}{v} \tag{6}$$

$$\tau_n m(k_0) = \tau_n(k_0) - \tau_m(k_0)$$

$$[\mathbf{W}_{io}]_{nm} = \exp(j2\pi f_i \tau_{nm}(\mathbf{k}_o)), \quad f_i = (i-1)F_s/N \tag{7}$$

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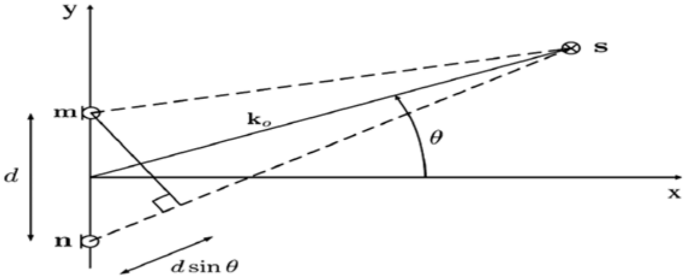

Figure: Array geometry consisting of two microphones m and n as seen from above, source azimuth angle given as θ.

Superposition of DoA Kernels:

- DoA kernels for each look direction O and at each frequency I is denoted by W_{io}
- source spatial image $S = H * \widehat{S}$

$$\mathbf{H}_{ik} = \sum_{o=1}^{O} \mathbf{W}_{io} z_{ko}, \qquad (8)$$

CNMF Model for SCM Observations:

$$\mathbf{X}_{il} \approx \hat{\mathbf{X}}_{il} = \sum_{k=1}^{K} \mathbf{H}_{ik}\hat{s}_{ilk} = \sum_{k=1}^{K}\sum_{o=1}^{O} \mathbf{W}_{io} z_{ko}\hat{s}_{ilk}. \tag{9}$$

$$\hat{s}_{ilk} = t_{ik}v_{kl}, \quad t_{ik}, v_{kl} \geq 0, \tag{10}$$

$$\mathbf{X}_{il} \approx \hat{\mathbf{X}}_{il} = \sum_{k=1}^{K}\sum_{o=1}^{O} \mathbf{W}_{io} z_{ko} t_{ik} v_{kl}. \tag{11}$$

$$\hat{\mathbf{X}}_{il} = \sum_{k=1}^{K} \mathbf{H}_{ik} t_{ik} v_{kl}. \tag{12}$$

CNMF Cost Function:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\|\mathbf{X}_{il}-\hat{\mathbf{X}}_{il}\|_F^2. \tag{13}$$

$$\mathcal{L}(\mathbf{W},\mathbf{Z},\mathbf{T},\mathbf{V})=\sum_{i=1}^{I}\sum_{l=1}^{L}\|\mathbf{X}_{il}-\sum_{k=1}^{K}\sum_{o=1}^{O}\mathbf{W}_{io}z_{ko}t_{ik}v_{kl}\|_F^2. \tag{14}$$

$$\mathbf{C}_{ilko}=\mathbf{W}_{io}z_{ko}t_{ik}v_{kl}+r_{ilko}(\mathbf{X}_{il}-\sum_{k,o}\mathbf{W}_{io}z_{ko}t_{ik}v_{kl}) \tag{15}$$

$$r_{ilko}=\frac{z_{ko}t_{ik}v_{kl}}{\hat{x}_{il}},\quad \hat{x}_{il}=\sum_{k,o}z_{ko}t_{ik}v_{kl}. \tag{16}$$

Algorithm Updates for the Non-negative Parameters:

$$z_{ko} \leftarrow z_{ko}\left[1+\frac{\sum_{i,l} t_{ik}v_{kl}\operatorname{tr}(\mathbf{E}_{il}\mathbf{W}_{io})}{\sum_{i,l} t_{ik}v_{kl}\hat{x}_{il}}\right] \tag{21}$$

$$t_{ik} \leftarrow t_{ik}\left[1+\frac{\sum_{l,o} z_{ko}v_{kl}\operatorname{tr}(\mathbf{E}_{il}\mathbf{W}_{io})}{\sum_{l,o} z_{ko}v_{kl}\hat{x}_{il}}\right] \tag{22}$$

$$v_{kl} \leftarrow v_{kl}\left[1+\frac{\sum_{i,o} z_{ko}t_{ik}\operatorname{tr}(\mathbf{E}_{il}\mathbf{W}_{io})}{\sum_{i,o} z_{ko}t_{ik}\hat{x}_{il}}\right]. \tag{23}$$

$$\mathbf{E}_{il} = \mathbf{X}_{il} - \sum_{k,o} \mathbf{W}_{io} z_{ko} t_{ik} v_{kl}$$

Algorithm Updates for the SCM Model Parameters:

$$\hat{\mathbf{W}}_{io} \leftarrow \mathbf{W}_{io}\left[\sum_{l,k} z_{ko}t_{ik}v_{kl}\hat{x}_{il} + \sum_{l,k} z_{ko}t_{ik}v_{kl}\mathbf{E}_{il}\right] \tag{24}$$

$$\hat{\mathbf{W}}_{io} \leftarrow \mathbf{V}\hat{D}\mathbf{V}^{H}. \tag{25}$$

$$\mathbf{W}_{io} \leftarrow |\hat{\mathbf{W}}_{io}|\exp(i\arg(\mathbf{W}_{io})), \tag{26}$$

$$\mathbf{W}_{io} \leftarrow \frac{\mathbf{W}_{io}}{\|\mathbf{W}_{io}\|_F} \tag{28}$$

$$\hat{a}_k = (\sum_{l=1}^{L} v_{kl}^2)^{1/2}, \quad v_{kl} \leftarrow \frac{v_{kl}}{\hat{a}_k}, \quad t_{ik} \leftarrow t_{ik}\hat{a}_k \tag{30}$$

$$\hat{b}_k = (\sum_{o=1}^{O} z_{ko}^2)^{1/2}, \quad z_{ko} \leftarrow \frac{z_{ko}}{\hat{b}_k}, \quad t_{ik} \leftarrow t_{ik}\hat{b}_k, \tag{31}$$

Algorithm Implementation:

1) Initialize z_{ko}, t_{ik} and v_{kl} and with random values uniformly distributed between zero and one.

2) Initialize W_{io} according to (7) and apply scaling (28).

3) Recalculate magnitude model $\hat{x}_{il}$ according to (16).

4) Update t_{ik} according to (22).l^2

5) Recalculate magnitude model $\hat{x}_{il}$ according to (16).

6) Update v_{kl} according to (23).

7) Scale v_{kl} to unity l^2-norm and compensate by rescaling $t_{ik:}$ as specified in (30).

8) Recalculate magnitude model $\hat{x}_{il}$according to (16).

9) Update z_{ko} according to (21).

10) Scale z_{ko} to l^2-norm and compensate by rescaling t_{ik} as specified in (31).

11) Recalculate magnitude model $\hat{x}_{il}$ according to (16).

12) Calculate $\widehat{W}_{io}$ according to (24) and enforce it to be positive semidefinite by (25).

13) Update W_{io} according to (26) and apply scaling (28).

14) The algorithm is repeating steps 3-13 for a fixed amount of iterations or until the parameter updates converge.

K mean:

- Apply k-means clustering on the spatial weights z_{ko},
- No of cluster is equal to the number of sound sources

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Source Reconstruction:

$$s_{ilq} = \sum_{ko} b_{qk} z_{ko} t_{ik} v_{kl}. \tag{32}$$

$$\mathbf{y}_{ilq} = \mathbf{x}_{il} \frac{\sum_{ko} b_{qk} z_{ko} t_{ik} v_{kl}}{\sum_{qko} b_{qk} z_{ko} t_{ik} v_{kl}}, \tag{33}$$

[1] J. Nikunen., "Direction of Arrival Based Spatial Covariance Mode for Blind Sound Source Separation",IEEE/ACM trans.., vol. 22, no. 3, p. 727, 2014.

[2] A. Ozerov. and C. Fevotte., "Multichannel nonnegativematrix factorization in convolutive mixtures for audio source separation",IEEE Trans. Audio, Speech, Lang. Process., vol. 18, no. 3, pp. 550563, Mar. 2010.

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